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Hypersonic nozzle flow of air with high initial dissociation levels

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An experimental investigation was undertaken of the flow produced by the inviscid expansion of air through a hypersonic nozzle. Stagnation enthalpy levels up to 4×10^7 J/kg were used, with initial dissociation levels approaching 90 %. By measuring the flow velocity and the frozen dissociation fractions of oxygen and nitrogen, it was found that existing theoretical models served adequately to define the nozzle flow, at least for the purpose of conducting experiments involving reacting inviscid hypersonic flows about blunt bodies.

1. Introduction

The steady expansion of air through a hypersonic nozzle is one of the most basic flows of gasdynamics. When the air is initially dissociated, thermochemical nonequilibrium effects are known to influence the composition and thermodynamic state of the expanded flow. Such effects were studied experimentally in shock tunnels by Nagamatsu & Sheer (1965), by Duffy (1965) and by Dunn (1969). These studies were limited to maximum stagnation enthalpies of $1{\cdot}6 \times 10^7$ J/kg and reservoir temperatures of 7200 °K. An attempt by Dunn to study the nozzle expansion at a stagnation enthalpy of $3{\cdot}3 \times 10^7$ J/kg was unsuccessful, as the nozzle reservoir pressure was limited to 3·8 atmospheres. This gave rise to strong viscous effects in the nozzle flow, which obscured the characteristics of the inviscid expansion.

The limitation experienced by Dunn has been removed by application of the free-piston driver technique to shock tunnels (Stalker 1967). This has made it possible to generate nozzle flows at stagnation enthalpies up to 4×10^7 J/kg, with reservoir temperatures of 12 000 °K and reservoir pressures exceeding 200 atm. These reservoir conditions allow temperatures and densities to be generated in the inviscid flow about a model in the test section which are such that it is possible to observe pronounced gasdynamic effects arising from chemical reactions. The reactions include the dissociation of nitrogen, indicating that laboratory studies may be performed on hypersonic reacting flows in air, at speeds up to and exceeding the earth satellite velocity. Therefore, not only is the study of an expanding nozzle flow intrinsically interesting, but it is also a matter of immediate practical interest to establish that it will behave as expected with these reservoir conditions, producing the predicted values of those free-stream properties which determine the structure of an inviscid flow field.

It should be noted that the normal 'wind tunnel' calibration of a shock tunnel is not adequate for this purpose. In such a case, it is generally intended that the free stream produced by the nozzle expansion should be used for high Mach number experiments involving surface measurements of pressure and heat transfer on moderately blunt aerodynamic configurations, i.e. on surfaces at incidences sufficiently large that they exhibit a thin shock layer. The stream is then adequately defined by combining measurements of Pitot pressure and stagnation-point heat transfer to specify the stream momentum and stagnation enthalpy. The nozzle flow in a free-piston shock tunnel has been studied previously from this point of view (Stalker 1967).

In general, a considerable number of free-stream properties would be required to determine the structure of a reacting inviscid flow field rigorously. Some of these (e.g. the temperature of the free stream) would be difficult to measure experimentally. Fortunately, the number can be reduced by noting that, in shock tunnel experiments, interest usually is centred on the flow field near a model, rather than far downstream. In this region the dissociation of oxygen and nitrogen will normally be the dominant chemical reaction, whilst the strength of the bow shock usually will be such that it can be treated according to the strong shock wave assumptions. It can then be shown that the flow about a model is determined by the free-stream density and velocity and the free-stream values of the dissociation fractions of oxygen and nitrogen.

It should be emphasized that there may be situations which are not covered by this specification of the free stream. For example, vibrational relaxation may occur or the presence of low concentrations of nitrous oxide may lead to reactions which are not accounted for. Such effects normally involve the exchange of relatively small amounts of energy with the flow, and will produce only small changes in the flow field in comparison with those produced by dissociation of oxygen or nitrogen. However, when such effects are an important feature of the model flow, the present free-stream specification will be inadequate.

In addition to free-stream specification for gasdynamic studies, a subordinate purpose of the experiments was to test the adequacy of existing models of the chemical kinetics of air for predicting the behaviour of an expanding stream tube. The experiments of Nagamatsu & Sheer, of Duffy and of Dunn had shown these models to be valid at stagnation temperatures up to 7200 °K. In particular, they confirmed the efficacy of the NO exchange reaction in promoting recombination of atomic nitrogen in the expansion (Eschenroeder, Boyer & Hall 1962). At the higher temperatures of the present investigation, numerical calculations (e.g. Harris & Warren 1964) indicated that the exchange reaction would fail to prevent the freezing of atomic nitrogen, and it was of interest to confirm this.

2. Measurement techniques

The determination of density and velocity is straightforward in principle, as the velocity can be measured directly by a tracer technique, and the density obtained by combining the velocity with a measurement of the Pitot pressure. However, a more indirect approach is required in order to determine the dissociation fractions in the free stream.

Measurement of the frozen density ratio across a shock wave was chosen for this purpose. Consideration of numerical solutions for the nozzle flow (Harris & Warren 1964) indicated that, as the stagnation enthalpy is increased, significant quantities of frozen nitrogen atoms do not appear in the free stream whilst molecular oxygen is present. This implies that the frozen dissociation fractions of both oxygen and nitrogen can be deduced from a knowledge of the ratio of the number of monatomic particles to the number of diatomic particles in the free stream. This ratio can be determined from the frozen ratio of specific heats γ by using the relation

$$N_1/N_2 = (5\gamma-7)/(5-3\gamma), \tag{1}$$

where N_1 is the number of monatomic particles and N_2 is the number of diatomic particles. γ can be determined from the expression for the frozen density ratio across a shock wave in the free stream, i.e.

$$(\gamma+1)/(\gamma-1) = (\rho_2/\rho_1)\,[1+2M_n^{-2}/(\gamma-1)], \tag{2}$$

where ρ_2/ρ_1 is the density ratio across the shock wave and M_n is the Mach number of the component of free-stream flow normal to the shock wave. In a hypersonic flow M_n will usually be sufficiently large that substantial errors can be made in estimating its value, without significantly affecting the accuracy of the equation. Indeed, in cases where M_n is particularly large, or some inaccuracy can be tolerated, (2) may more conveniently be written in the approximate form

$$(\gamma+1)/(\gamma-1) = \rho_2/\rho_1. \tag{2a}$$

In the present experiments, the shock wave was produced by a two-dimensional wedge with a semi-vertex angle of 35°. The density ratio across the shock was determined from schlieren photographs by measuring the angle between the wedge surface and the shock and using the relation

$$\rho_1/\rho_2 = \tan\delta/\tan(\theta+\delta), \tag{3}$$

where δ is the angle between the shock and the wedge surface, and θ is the angle made by the wedge surface with the direction of the free stream. This configuration had the advantage that the angle between the shock wave and the free-stream direction was sufficiently low that temperatures high enough to promote significant dissociation reactions downstream of the shock wave were avoided at high stagnation enthalpies. At the same time, because the semi-vertex angle was within a few degrees of the shock detachment angle, δ was relatively sensitive to changes in the density ratio. Indeed, it can be shown that measurement of δ with an accuracy of $\pm 0{\cdot}5°$ at high enthalpies is sufficient to determine the free-stream nitrogen dissociation fraction to within $\pm 0{\cdot}08$. This had the disadvantage that a number of corrections to the simple relation implied by (3) were necessary in interpreting the results. However, these were such that they could be applied with confidence, at least above nominal stagnation enthalpy levels of $1{\cdot}5\times10^7$ J/kg.

It may be noted that the free-stream static pressure has been used as an indicator of non-equilibrium effects in previous studies. However, numerical computations indicated that the static pressure varied by only $\pm 20\,\%$ as the stagnation enthalpy increased from $1{\cdot}5\times10^7$ to 5×10^7 J/kg, implying that the stagnation enthalpy must itself be known before static pressure measurements can be interpreted. This is difficult to obtain accurately. Whilst measurements in the shock tube provide a nominal value, the possibility of effects such as mixing of the shock-tube boundary layer into the test gas by shock reflexion, and radiative energy loss at high stagnation enthalpies, make it unreliable as an absolute measurement. In these experiments, the main contribution to the stagnation enthalpy was derived from the velocity measurements, and there was an error of $\pm 10\,\%$ from this source. This is reflected in errors in the free-stream composition. For example, at a nominal stagnation enthalpy of 3×10^7 J/kg, an error of $\pm 10\,\%$ would imply an uncertainty of $\pm 0{\cdot}07$ in the absolute value of the free-stream nitrogen dissociation fraction. This is comparable with that obtained by the wedge method. Therefore, as the wedge method was also relatively simple to employ, it was preferred for these experiments.

The wedge method could also be made to serve another purpose. In shock tunnel operation the mixing processes resulting from the shock–boundary-layer interaction associated with shock reflexion at the downstream end of the shock tube (Davies & Wilson, 1967) lead to early contamination of the test gas with driver gas, and it is important to determine at what stage this seriously affects the flow. As helium was used as the driver gas in these experiments, its appearance in the free stream was evidenced by an increase in N_1, and therefore by an increase in the shock angle. Thus, by monitoring the variation of shock angle with time, the useful test period could be defined.

3. Experiments

The experiments were conducted in a small free-piston shock tunnel. The compression tube was 1·5 m long and 50 mm diameter, whilst the shock tube was 1·4 m long and 12 mm in diameter. This was attached to hypersonic nozzle with a throat diameter of 1·65 mm. An axisymmetric nozzle with an exit diameter of 75 mm was used for measurements of test section velocity. However, the test section densities obtained with this nozzle were too low to allow effective use of the schlieren technique, and therefore a conical nozzle with an exit diameter of 38 mm and an included divergence angle of 15° was used for the shock angle measurements.

During the course of the experiments, shock speeds were monitored by using thin-film heat-transfer gauges, or alternatively, piezoelectric pressure transducers, stationed along the tube. Shock attenuation was measured from a series of luminosity streak photographs. These were obtained by inserting a transparent section 0·5 m long at the downstream end of the shock tube, and viewing the attenuation with an STL image converter camera. The streak records showed that the shock speed attenuated by approximately 25 % per metre over the last 0·5 m of the tube, and to take account of this, the value 0·3 m from the downstream end was used in all calculations.

The pressure at the end of the shock tube, after shock reflexion, was monitored with an S.L.M. type HPZ-14 piezoelectric pressure transducer. This was mounted in the side wall of the shock tube, 7·5 mm from the downstream end.

Wave reflexions, between the end of the shock tube and the contact surface, gave rise to an initial period of unsteadiness in the pressure. However, this generally lasted for less than 50 μs, and was followed by at least 200 μs of reasonably steady pressure, which was taken as the nozzle reservoir pressure. At stagnation enthalpies in excess of 2×10^7 J/kg, this pressure was 230 atm, whilst at lower stagnation enthalpies it was reduced to 100 atm.

The measured shock speeds were consistent with early results obtained with the apparatus, which have been reported previously (Stalker 1965). They were always less than, or equal to, the tailored interface value. The nominal stagnation enthalpy, which henceforth will be referred to simply as the stagnation enthalpy, was calculated for each test by using the shock speed and the initial pressure in the shock tube to yield the pressure and enthalpy immediately after shock reflexion, and then assuming isentropic expansion to the measured nozzle reservoir pressure.

The free-stream velocity was measured by a spark tracer technique, by observing the downstream motion of a column of the test gas which was initially heated by a spark between the tips of two electrodes in the test section. This technique is discussed in a previous paper by McIntosh (1971), where it has been found to yield results which are consistent with those obtained by a magnetohydrodynamic method. In the present experiments, the electrode tips were 42 mm apart on a line perpendicular to the nozzle axis approximately 17 mm downstream of the nozzle exit. The spark, which was produced by discharge of a condenser of capacity 0·004 μF initially charged to 9 kV, was of very short duration compared with the test section flow transit time. The downstream motion of the resulting luminous column of test gas was traced using the image converter camera, operating in the streaking mode.

The main source of error in the measurements was expected to originate in reading the photographic streak records of the gas motion. These were obtained using Polaroid type 410 film, and by averaging the values obtained from the print and the negative in each test, an estimated accuracy of $\pm 5\%$ was achieved.

Pitot pressures were measured using piezoelectric pressure transducers. With careful attention to mounting of the transducers, an estimated measurement accuracy of $\pm 7\%$ could be obtained. In order to isolate the transducers from thermal and electrical effects originating from contact with the hot test gas, they were protected by a small heat exchanger. By minimizing the size of the heat exchanger, response times of less than 50 μs could be obtained. Pitot surveys were conducted in the nozzle which was used for the shock angle measurements, and the results of such a survey are displayed in figure 1. This shows the existence of a uniform test core at least 20 mm in diameter and an axial pressure gradient corresponding to an included nozzle divergence angle of $11 \pm 2°$.

For the shock angle measurements, a symmetrical hardened steel wedge 18 mm thick and 25 mm wide was used. The angle between the wedge faces was 70°, and each wedge face was 16 mm long in the streamwise direction. Schlieren photographs were taken using both single-pass and double-pass systems, illuminated by a short-duration ($< 2\,\mu$s) spark light source. The light source was triggered, with a suitable time delay, from the signal generated by the HPZ-14

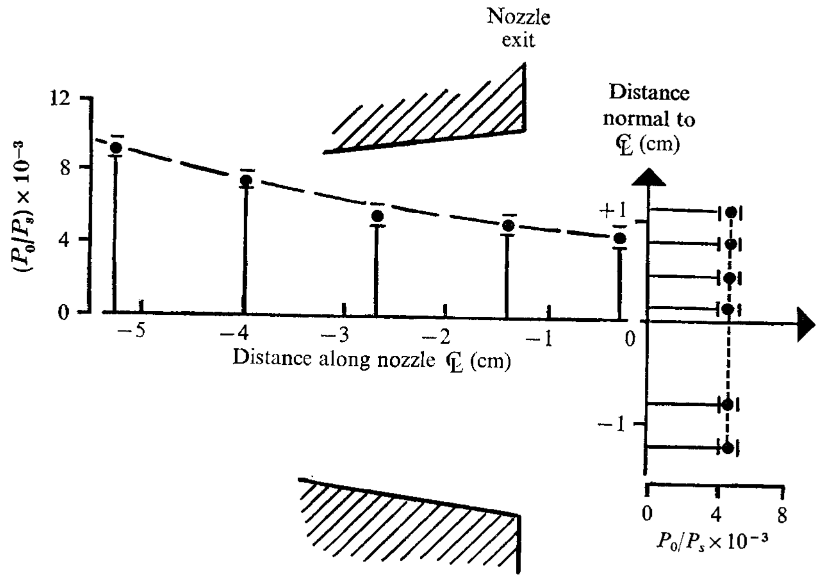

FIGURE 1. Pitot pressure survey. ---, expected axial variation, 11° nozzle divergence; $\bar{\bullet}$, experiments. P_0, pitot pressure; P_s, nozzle reservoir pressure (= 220 atm); stagnation enthalpy = 4×10^7 J/kg.

pressure transducer upon shock reflexion. To eliminate luminosity effects arising from the hot gas flowing over the wedge, a masking slit was used at the second knife edge, together with a Wratten type 47 filter. The shock angles were measured at a distance along the shock wave approximately 13 mm from the leading edge, with an estimated accuracy of $\pm 0{\cdot}5°$.

4. Results and discussion

Contamination by driver gas and test period

As was outlined in §2, the onset of driver-gas contamination could be detected by monitoring the variation of shock angle with time. This was done, for each of a number of different test conditions, by varying the time delay of the schlieren light source over a series of tests. Examples of such measurements are shown in figure 2. For the results shown in the figure, the shock-tube driver conditions remained constant, and the nozzle stagnation enthalpy was varied by changing the initial pressure in the shock tube. The data at the highest and the lowest enthalpy levels on the figure shows that, following an initial starting period of less than 60 μs duration, the value of the shock angle remains at a constant 'plateau' level for a time, and subsequently rises. This rise is ascribed to contamination by the helium driver gas. For the conditions of figure 2, an increase of 0·5° in the shock angle corresponds to a helium contamination level of approximately 15 % by volume, or 3 % by mass. At the highest stagnation enthalpy on the figure (4×10^7 J/kg), the shock tunnel was operating close to the tailored interface condition, and therefore these tests would be expected to display the earliest evidence of contamination. This is borne out by comparison with the

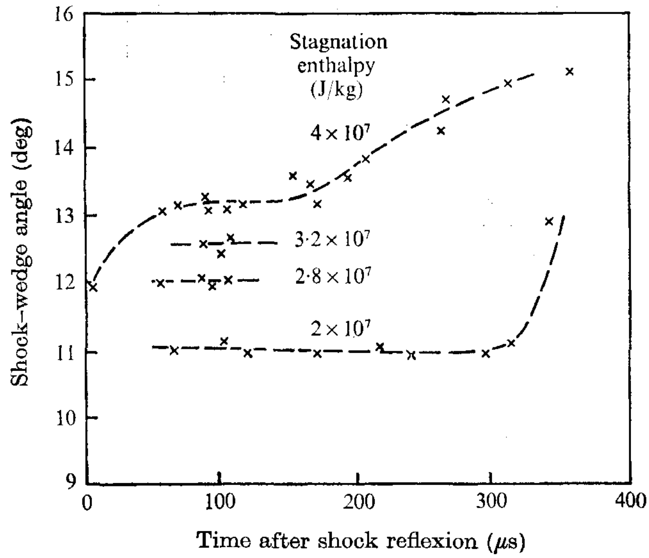

FIGURE 2. Variation of shock-wedge angle with delay time. $\times$, experiments; --, faired curves.

results at the lowest enthalpy (2×10^7 J/kg), where operation at a shock speed well below the tailored interface value has produced a considerable delay in the onset of contamination. At the two intermediate enthalpy values it can safely be concluded that the results are free of contamination for at least as long a period as the high enthalpy results.

By using this method, it was established that a useful period of steady flow existed at all test conditions. This was further confirmed by time-resolved measurements of stagnation-point heat transfer on a hemispherically blunt body. The steady flow period began within 100 μs of shock reflexion, and exhibited the shortest duration at the highest stagnation enthalpy. In this case, it can be seen from figure 2 that it lasted for 80 μs.

Velocity and density measurements

The velocity measurements were taken during the test period by triggering the spark gap in the test section between 110 μs and 150 μs after shock reflexion. The results obtained are presented in figure 3, and are seen to compare satisfactorily with values predicted using the nozzle program of Lordi, Mates & Moselle (1966).

Pitot pressure measurements were made for all test conditions, and it was found that the ratio of Pitot pressure to nozzle reservoir pressure did not vary with stagnation enthalpy by more than 25 %. By combining the estimated error in Pitot pressure with the error in velocity, an error of ± 17 % in the free-stream density was obtained.

Shock angle on wedge

As may be seen from figure 2, the angle between the shock wave and the wedge, during the steady flow period, could be readily measured at each test condition. The resultant values are plotted against stagnation enthalpy in figure 4.

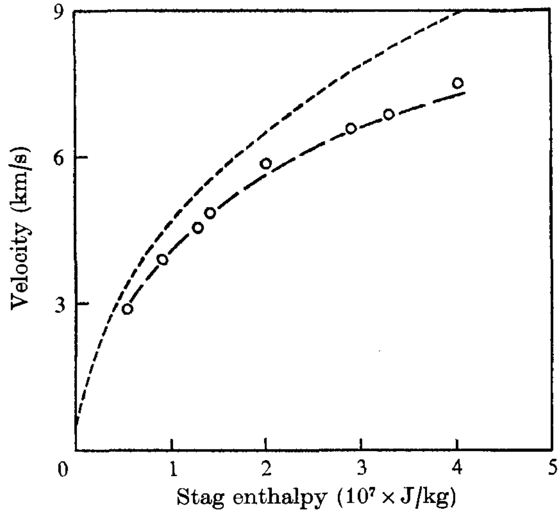

FIGURE 3. Velocity measurements. ○, experiments; ––, theory (Lordi *et al.* 1966); ------, ($2 \times$ stagnation enthalpy)$^{\frac{1}{2}}$.

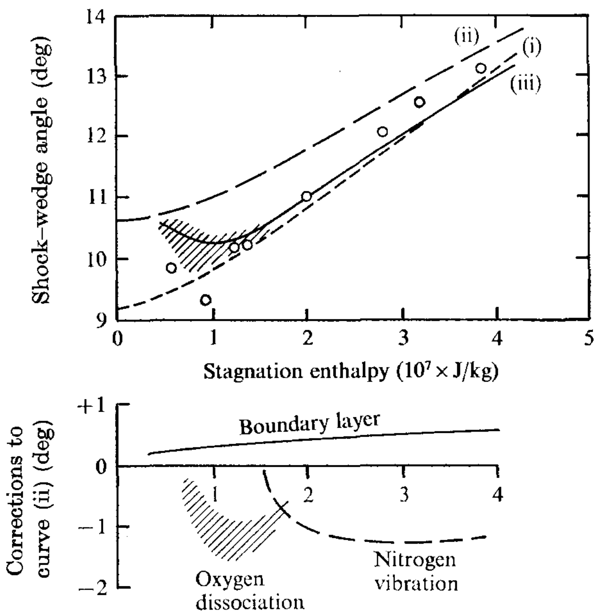

FIGURE 4. Steady flow shock–wedge angle. ○, experiment. Predicted shock–wedge angle: (i) first approximation; (ii) corrected for flow divergence and finite M; (iii) also corrected for boundary-layer and relaxation effects.

These measurements may readily be interpreted to yield the free-stream dissociation fractions if the free-stream flow is uniform, the free-stream Mach number is very large, the Reynolds number is sufficiently large that the boundary layer formed on the surface of the wedge does not affect the shock wave, and the densities and temperatures are sufficiently low that no chemical or vibrational relaxation occurs as the gas flows over the wedge. Under such conditions, (1), (2*a*) and (3) may be used to calculate the shock angle directly from those free-stream dissociation fractions which are predicted by the nozzle flow computer program. This yields curve (i) shown in figure 4. It is seen that, except at low enthalpies, the curve agrees with the experimental results, predicting not only the variation in shock angle with stagnation enthalpy, but also the absolute value.

However, the agreement in absolute value is fortuitous. For the experiments none of the four conditions noted above is satisfied and, in each case, this modifies the absolute value of the shock angle. As will be seen below, agreement arises because the consequential corrections to the shock angle largely cancel each other when they are all taken together.

In considering the corrections, it is convenient to consider the four effects in order, beginning with the free-stream flow non-uniformity. This arose from the flow divergence associated with the conical nozzle. Using the theoretical treatment of Hall (1963), and the effective flow divergence angle of 11° obtained from the Pitot pressure surveys, the effect of flow divergence on the shock shape could be calculated. Typically, at 13 mm from the leading edge of the wedge, this tended to reduce the shock angle by 1·7° at a stagnation enthalpy of 3×10^7 J/kg. The experimental error in the flow divergence angle contributed an uncertainty of $\pm 0{\cdot}3°$ to this shock angle reduction.

The Mach number correction appears in equation (2). The value of M_n, for use in the equation, was obtained from the nozzle-flow computer program, and was typically about 5·5. This value was indirectly confirmed by conducting a series of tests with a wedge of vertex angle 50°, together with a nozzle throat of diameter 3·2 mm.† This reduced M_n to values near 3·5, and caused an increase in the angle between the shock and the wedge of roughly 25 %. Observation of the expected increase was accepted as a check on the magnitude of the Mach number correction. When this correction, together with that due to flow divergence, is applied to curve (i) in figure 4, curve (ii) results.

The boundary-layer correction was calculated by obtaining the displacement thickness for a frozen zero-pressure-gradient boundary layer with Prandtl and Lewis number each equal to unity, and the product of density and viscosity constant at a value given by the reference enthalpy (Hayes & Probstein 1959, p. 296). These assumptions were substantiated by comparing boundary-layer density profiles with approximate profiles obtained with a Mach Zehnder interferometer in other experiments. As shown in figure 4, it was found that the boundary-layer correction tended to increase the shock angle by 0·5° at the highest stagnation enthalpy, and 0·2° at the lowest.

To assess possible effects of chemical reactions on the shock angle, calculations were performed with the assumption of constant pressure downstream of the shock wave. Where the temperature behind the shock wave did not exceed 8000 °K, published reaction rates for the components of air (Prud'Homme &

† With this throat, it was necessary to attach the nozzle to a larger shock tube, in order to provide a supply of test gas sufficient to allow a reasonable test time. However, the shock tube was operated with the same initial pressure and shock speeds as before, producing identical reservoir conditions for the nozzle flow.

Lequoy 1969) were used. This corresponded to stagnation enthalpies below 2×10^7 J/kg and in this range, it was found that oxygen dissociation was a significant factor in determining shock angles. For stagnation enthalpies above this level, initial calculations were done with the same reaction rates extrapolated to the higher temperatures prevailing. However, this indicated that dissociation of nitrogen via the shuffle reactions, $N_2 + O \rightarrow NO + N$ and $NO + M \rightarrow N + O + M$, would seriously influence the shock angle, producing values which could be of the order of 2° less than those observed.

To check this, a series of experiments was performed to observe the effects of these reactions in the high temperature region produced by a normal shock in the test section. The shock was formed near the stagnation point of a circular cylinder, 19 mm in diameter, which was aligned at right angles across the flow. The Mach Zehnder interferometer was employed to measure the density and the rate of density change in the region between the bow shock and the cylinder. In order to produce easily measurable fringe shifts, the 3·2 mm diameter nozzle throat was used. It was convenient to express the results in terms of the rate of proportional density change, divided by the absolute density, i.e. $\rho^{-1} d(\ln \rho)/dt$. This had a value of 10^6 m^3 kg^{-1} s^{-1} at a stagnation enthalpy of $1{\cdot}6 \times 10^7$ J/kg and increased, in a roughly linear fashion, to 5×10^6 m^3 kg^{-1} s^{-1} at $3{\cdot}6 \times 10^7$ J/kg. By assuming that binary scaling applied, these results could be used to estimate an upper limit to the effect of the above reactions in the flow downstream of the oblique shock. The estimates showed that, when some allowance was made for the reduced temperatures behind the oblique shock, the shuffle reactions did not significantly influence the shock angle.

It is perhaps worth noting that, in the above experiments, the temperatures and species concentrations in the reaction zone could be obtained by using the measured free-stream velocities, and anticipating the final results of this paper for the species concentrations. The temperatures immediately downstream of the normal bow shock were then seen to range from 10 000 to 18 000 °K; that is, temperatures lay outside the range for which existing formulae for reaction rates have been derived (Prud'Homme & Lequoy 1969). Comparison of the observed values of the rate of density change with those calculated from these formulae showed that agreement prevailed at the lower temperature, but that at 18 000 °K the calculated value was roughly an order of magnitude too high. Whilst it cannot be said at this stage whether or not this is a tunnel effect (arising, for example, from the non-equilibrium state of the free stream) it does suggest a need for better understanding of the factors influencing the shuffle reactions at temperatures over 10 000 °K.

Whilst the above experiments showed that chemical reactions could be neglected at stagnation enthalpies above 2×10^7 J/kg, the test conditions were such that it was necessary to take account of the vibrational relaxation of nitrogen molecules. This could have been done by following procedures outlined in previous studies of the non-equilibrium flow over a wedge with a strong attached shock wave, involving either numerical techniques (e.g. Sedney, South & Gerber 1964; Capiaux & Washington 1963) or perturbation analyses (e.g. Lee 1964; Zhigulev 1962). However, it was convenient to make use of an available computer program,† which allowed calculation of the flow field associated with a given shock shape for a gas which was undergoing both chemical and vibrational relaxation. The program was executed on a Univac 1108 computer, using vibrational relaxation rates obtained from Millikan & White (1963), with the effect of upstream vibrational 'freezing' incorporated by appropriately reducing the maximum allowable vibrational energy downstream of the shock wave. For computations at low stagnation enthalpies, the reaction rates for oxygen listed by Prud'Homme & Lequoy (1969) were used.

It might be noted that the need to take account of the frozen vibrational energy of nitrogen in the free stream introduced a factor which was based on theoretical extrapolation of lower enthalpy results (Hurle, Russo & Hall 1964) and, unlike the other high enthalpy corrections to the wedge angle, has not been checked by experiments. However, in order to affect the shock angle significantly, the extrapolated vibrational relaxation rates in the nozzle flow would need to be in error by two orders of magnitude, and this seems improbable.

As shown in figure 4, it was found that vibrational relaxation caused a reduction in shock angle of approximately $1{\cdot}3^\circ$, which persisted down to a stagnation enthalpy of $1{\cdot}8 \times 10^7$ J/kg, and rapidly disappeared with further reductions in enthalpy. A change by a factor of two in the relaxation rate did not significantly alter the effect. At enthalpies below 2×10^7 J/kg, the expected influence of oxygen dissociation appeared. It was found that, below $1{\cdot}6 \times 10^7$ J/kg, the calculations were sensitive both to the assumed reaction rates for oxygen and the experimental error ($\pm 5\%$) in the free-stream velocity. This led to uncertainty in the predicted shock angle at these enthalpy levels.

The corrections due to the boundary layer, the vibrational relaxation of nitrogen and oxygen dissociation were applied to curve (ii) in figure 4, to produce curve (iii). Noting that the uncertainties mentioned above render some of the calculations for oxygen unreliable, the portion of the curve (iii) thus affected is overlaid with cross-hatching.

The shock angle measurements could then be used to obtain the free-stream dissociation fraction by subtracting the difference between curve (iii) and curve (i) from the measured shock angle, and applying equations (3), (2 *a*) and (1) to the resultant shock angle. The values obtained are shown in figure 5. Error bars corresponding to an error of $0{\cdot}5^\circ$ in the shock angle are shown. Results which are substantially influenced by oxygen dissociation have been omitted. Curves for the predicted dissociation fractions (Lordi *et al.* 1966) are also displayed, and it can be seen that the experimental results are consistent with these curves.

5. Conclusion

The experiments established that the expansion of air in a hypersonic nozzle follows theoretical predictions up to a stagnation enthalpy of 4×10^7 J/kg, producing expected values of the velocity and the free-stream dissociation fractions of oxygen and nitrogen. The free-stream dissociation fractions were measured by

† This program was adapted from a program for calculation of blunt-body flows due to Garr & Marrone (1963). We thank H. Hornung for making the program available.

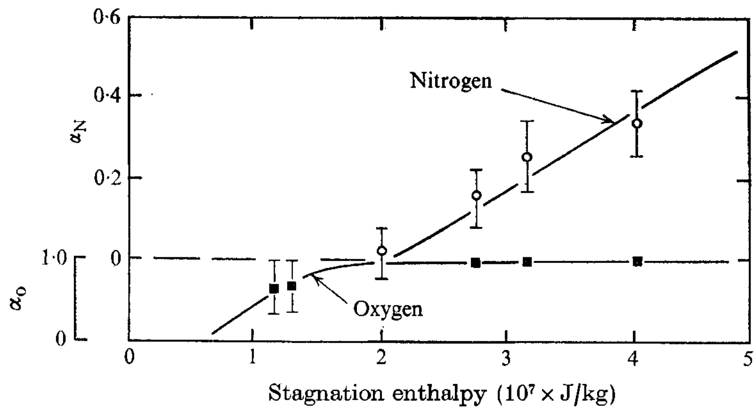

FIGURE 5. Free-stream dissociation fractions. Experiments: ○, nitrogen; ■, oxygen. ——, theory (Lordi *et al.* 1966).

$$\alpha_N = \frac{\text{Mass of nitrogen atoms/unit volume}}{\text{Total mass of nitrogen/unit volume}},$$

$$\alpha_O = \frac{\text{Mass of oxygen atoms/unit volume}}{\text{Total mass of oxygen/unit volume}}.$$

using the frozen density ratio across a shock wave, as obtained from shock angle measurements on a wedge. The use of a wedge presumed knowledge of the approximate level of free-stream vibrational excitation in nitrogen. Also, it was very inaccurate below stagnation enthalpy levels of $1{\cdot}5 \times 10^7$ J/kg, owing to effects of oxygen dissociation, but because this range has been covered by other investigators using static pressure measurements, the omission is not serious.

Remembering that the primary purpose of the experiments was to define the hypersonic flow produced by a nozzle, for the purpose of studying model flows involving chemical reactions, the effect of the errors in the measured quantities on the reaction rate immediately downstream of a shock wave is particularly significant. This may be estimated by interpreting the error in the velocity measurement in terms of the error in temperature downstream of the shock wave, and combining this with the errors in density and free-stream dissociation fraction. Typically, at a stagnation enthalpy of 3×10^7 J/kg, it is found that these errors contribute a factor of 2 to the uncertainty in the dissociation rate of nitrogen downstream of a normal shock. In a recent paper on reacting blunt-body flows, Hornung (1972) has estimated that a change in reaction rate by a factor of 2 would just produce detectable changes in the density field downstream of the bow shock wave, indicating that the accuracies achieved in the present experiments are adequate for defining the free stream for this purpose.

However, the accuracy with which the free stream is defined is not adequate for some flows involving relatively weak effects due to chemical reactions. For example, Kewley (1971) has recently studied flows over wedges (which were an order of magnitude longer than those studied here), observing the shock curvature due to nitrogen dissociation. He estimated that changes of 50 % in the reaction rate would produce detectable effects in the shock curvature, whilst the errors noted above would produce a factor of uncertainty of three in the nitrogen dissociation rate. Therefore such experiments probably will require an improvement in the accuracy with which the free stream is defined. As the main source of error stems from the terms involving temperature in the expressions for the reaction rates, it appears that such improvement might best be sought by improving the accuracy of the velocity measurements.

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